Reply to `critical properties of the *XXZ* chain in an external staggered magnetic field'

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COMMENT

Reply to 'critical properties of the XXZ chain in an external staggered magnetic field'

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Abstract. We discuss the comments presented by Okamoto and Nomura about the phase diagram of the XXZ chain under the influence of an external staggered magnetic field, reported by Alcaraz and Malvezzi.

In [2] the authors, although agreeing with our schematic phase diagram presented in [1] of the XXZ chain in external staggered magnetic field $H(\Delta, 0, h_s)$ (equation (1) of [1]), pointed out that the phase transition curve separating the massless phase (ML) and the antiferromagnetic phase (AF) (see figure 3 of [1]) is of infinite order. We agree with this possibility since from our analysis in [1] the mass-generating mechanism for $h_s \approx 0$ is the same as that of the sine–Gordon model. Actually, this is the conjectured result predicted earlier by den Nijs [3], by exploring the connection between the eight-vertex and the Gaussian model.

The phase transition curve separating the (ML) and (AF) phases was estimated by solving the phenomenological renormalization-group equation

$$MG_M(\Delta, h_s) = (M-2)G_{M-2}(\Delta, h_s) \tag{1}$$

where $G_M(\Delta, h_s)$ is the gap of the Hamiltonian with M sites. In [2] the authors point out the difficulty in using (1) in infinite-order phase transitions. Actually, the difficulty in using (1) is not the infinite-order nature of the phase transition, but the fact that in the whole (ML) phase the correlation length $\xi \sim M$. This is a well known problem and was discussed earlier by several authors [4–6]. We do not mean in [1] that by using (1) we have a precise evaluation of the curve separating the (ML) and (AF) phases. Our intention in using (1) in [1] was just to give an additional argument in favour of the conjecture which states the anisotropy $\Delta = \sqrt{2}/2$ as the starting point ($\sqrt{2}/2 \leqslant \Delta \leqslant 1$) where the (ML) appears.

A different method we can use to estimate this phase transition curve is by exploiting the fact that in the whole (ML) phase we have a Gaussian phase with conformal central charge $c = 1$. From conformal invariance (see equation (5) of [1]) the ratio of gaps

$$R_M(\Delta, h_s) = \frac{E_M^{(2)}(\Delta, h_s) - E_M^0(\Delta, h_s)}{E_M^{(1)}(\Delta, h_s) - E_M^0(\Delta, h_s)} \approx 4(1 + aM^{2-\bar{x}}) \tag{2}$$

should tend to 4 as $M \to \infty$, in the (ML) phase. In equation (2) $E_M^{(i)}(i = 0, 1, 2)$ is the lowest eigenenergy of the Hamiltonian $H(\Delta, 0, h_s)$ in the sector with z-magnetization $\frac{1}{2}\sum \sigma_i^z = i$. The constant a in (2) is lattice-independent and $\bar{x}$ is the dimension of the operator governing

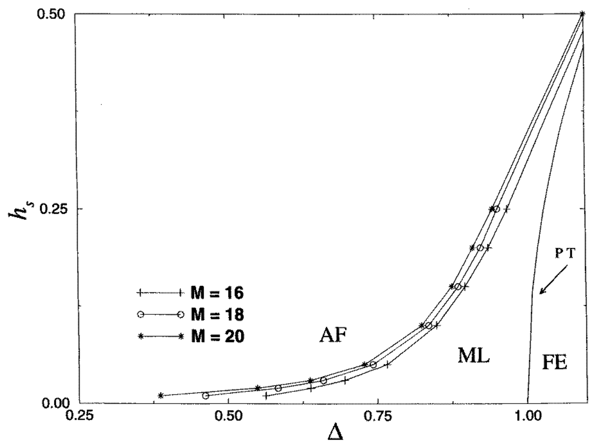

Figure 1. Finite-size estimators of the phase diagram of the XXZ chain in external staggered magnetic field. The anisotropy is Δ and h_s is the staggered field. The finite size estimates were obtained by solving the equation $R_M(\Delta, h_s) = \delta_M = 3.98$, for $M = 16$, 18 and 20.

the finite-size corrections [8]. Since R_M approaches 4 from below in figure 1 we plot the curve obtained from the equation $R_M(\Delta, h_s) = \delta_M = 3.98$ as a finite-size estimate of the transition curve, for $M = 16$, 18 and 20. These results are similar to those of figure 3 of [1], obtained by using equation (1). Actually, in the equation $R_M(\Delta, h_s) = \delta_M$, we should use different values of δ_M, for different lattice sizes (closer to 4 as M increases). This is the reason why the finite-size estimates are in the wrong relative positions for $\Delta \leqslant 0.75$ and small values of h_s.

In summary our results of figure 3 of [1] obtained by using (1) are equivalent to the results of figure 1 obtained by exploiting the relation (2) which together with the overall analysis presented in [1] strongly suggest the point $\Delta = \sqrt{2}/2$ as the starting point where the perturbation produced by the staggered magnetic field becomes irrelevant, producing the (ML) phase.

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